and longitudinal spacing of the vortex street, respectively, mm; c_V, vortex velocity, m/sec; Γ, vortex circulation, m^2/sec; $\bar{a}$, amplitude of oscillogram of density-gradient pulsations in a wake with a body behind a model; a_0, amplitude of oscillogram of density-gradient pulsations in an unperturbed wake; γ, intermittence factor of the vortex street structures; T, duration of continuous vortex street, sec; ΣT, total time of recording of vortex convergence process, sec; n, frequency of vortex convergence, 1/sec; c, flow velocity, m/sec; $\sqrt{\bar{p}^2}$, mean amplitude of pulsations in base pressure of model with a body in its wake, Pa; $\sqrt{\bar{p}_1^2}$, mean amplitude of pulsations of base pressure of model with an unperturbed wake, Pa; P_b, base pressure of model with a body in its wake, Pa; P_{b0}, base pressure of model with an unperturbed wake, Pa; $\sqrt{\bar{p}_0^2}$, mean amplitude of pressure pulsations at the forward stagnation point of a cylinder located in the model wake, Pa; $\sqrt{\bar{p}_{00}^2}$, mean amplitude of pressure pulsations at the forward stagnation point of a cylinder located outside the model wake, Pa; x_*, distance between model and a body in its wake corresponding to a resonance increase in pulsation amplitude, mm; a, speed of sound, m/sec; M, Mach number; $Re = Lc/\nu$, Reynolds number; $Sh = nH/c$, Strouhal number; $A = \sqrt{\bar{p}^2}/\sqrt{\bar{p}_1^2}$, relative amplitude of base-pressure pulsations of the model; $A_1 = \sqrt{\bar{p}_0^2}/\sqrt{\bar{p}_{00}^2}$, relative amplitude of pressure pulsations at the forward point of the cylinder; $\varepsilon = p_b/p_{b0}$, relative base pressure of model.

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D. B. Vafin, A. F. Dregalin, and A. B. Shigapov

UDC 536.3

Results are presented of a calculation of the heat radiation from a two-phase mixture in Laval nozzles in unidimensional and two-dimensional formulations.

The motion of a two-phase mixture in curved channels such as are present in Laval nozzles is characterized by substantial longitudinal and transverse gradients of the gasdynamic parameters in the transonic and supersonic flow regions. The radiation properties of both the gas phase and the particles of the condensed phase depend on the gasdynamic and thermodynamic characteristics of the medium. As a result, significant optical discontinuities occur both along and across the flow in Laval nozzles. Certain studies conducted in a two-dimensional approximation [1-3] show that errors may result from calculating the radiation from two-phase media in a unidimensional formulation of the problem of radiative heat transfer in the presence of substantial optical discontinuities or without allowance for the actual shape of the radiating volume.

Described below is a method of calculating the heat radiation from two-phase flows in axisymmetrical volumes with smooth diffuse-reflecting and radiating sides of arbitrary form. To describe the radiant energy transfer process, we used two-dimensional equations of the P_1-approximation of the spherical harmonics method. Calculations in a unidimensional formulation were performed in the P_3-approximation for an infinite cylinder.

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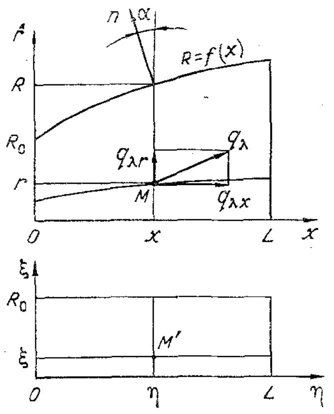

Fig. 1

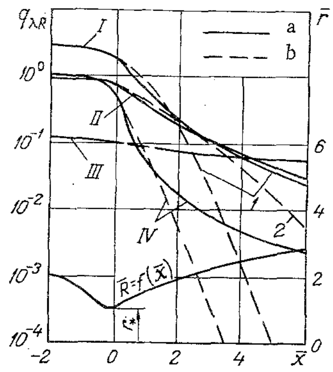

Fig. 2

Fig. 1. Chosen coordinate system.

Fig. 2. Distribution of flux densities for flows of thermal radiation incident on the nozzle wall: $r_* = 0.15$ m; a) two-dimensional problem; b) infinite cylinder (P_3-approximation); $r_W = \varepsilon_W = 0.5$; $T_W = 1000°K$ (I – $\lambda = 1\,\mu$m; II – 2; III – 4; IV – 0.5 μm). $q_{\lambda R}$, MW/m$^2\cdot\mu$m.

Figure 1 shows the diverging part of the Laval nozzle and the coordinate system for the problem being examined. The region of integration is given by the equation of the generatrix $R = f(x)$, the minimum cross section R_0, and the nozzle length L.

Using the P_1-approximation, we can obtain a system of differential equations in partial derivatives describing radiant energy transfer in the axisymmetrical absorbing and anisotropically scattering volumes:

$$\frac{\partial q_{\lambda r}(r,\ x)}{\partial r}+\frac{\partial q_{\lambda x}(r,\ x)}{\partial x}+\frac{q_{\lambda r}(r,\ x)}{r}+\alpha_\lambda(r,\ x)\,c_\lambda U_\lambda(r,\ x)=\eta_{c\lambda}(r,\ x),$$

$$c_\lambda\frac{\partial U_\lambda(r,\ x)}{\partial x}+3\,K_{1\lambda}(r,\ x)\,q_{\lambda x}(r,\ x)=0, \tag{1}$$

$$c_\lambda\frac{\partial U_\lambda(r,\ x)}{\partial r}+3\,K_{1\lambda}(r,\ x)\,q_{\lambda r}(r,\ x)=0.$$

System (1) is supplemented by the following boundary conditions on the symmetry axis

$$\frac{\partial U_\lambda(r,\ x)}{\partial r}=0 \tag{2}$$

and on the bounding surfaces

$$c_\lambda(1-r_w)\,U_\lambda(r,\ x)+2\sin\alpha\,(1+r_w)\,q_{\lambda x}(r,\ x)-2\cos\alpha\,(1+r_w)\,q_{\lambda r}(r,\ x)=4\,\pi\varepsilon_w I_{\lambda b}(T_w)\quad\text{at}\quad r=R,$$

$$c_\lambda(1-r_w)\,U_\lambda(r,\ x)+2\,(1+r_w)\,q_{\lambda x}(r,\ x)=4\,\pi\varepsilon_w I_{\lambda b}(T_w)\quad\text{at}\quad x=0, \tag{3}$$

$$c_\lambda(1-r_w)\,U_\lambda(r,\ x)-2\,(1+r_w)\,q_{\lambda x}(r,\ x)=4\,\pi\varepsilon_w I_{\lambda b}(T_w)\quad\text{at}\quad x=L.$$

Boundary conditions (3) were obtained in the form of conditions for diffuse-reflecting and radiating nonconcave surfaces in [4].

With the assignment of the temperature field and radiation characteristics of the medium and bounding surfaces, system (1) together with the boundary conditions (2) and (3) unambiguously define the flux density of the spectral radiation $q_\lambda(r,\ x)$.

To construct a difference grid, we connect the position of the boundaries with a curvilinear coordinate system in which the boundaries of the region are the coordinate lines. Then, in the curvilinear system of coordinates $(\xi,\ \eta)$, the region is a rectangle (Fig. 1).

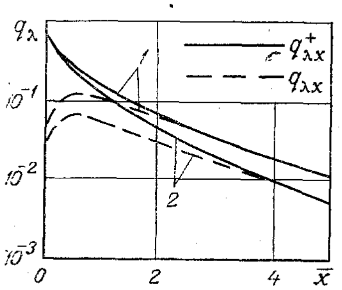

Fig. 3. Change in flux densities in the axial direction: 1, 2) $r_* =$ 0.1 and 0.2 m, respectively; $r_w = \varepsilon_w = 0.5$; $T_w = 1000°K$; $\lambda = 0.5\,\mu m$; q_λ, $MW/m^2 \cdot \mu m$.

The equations which transform the region of integration into a rectangle are

$$\xi = \frac{rR_0}{f(x)}, \quad \eta = x. \tag{4}$$

Using the above relations, we can write system (1) in curvilinear coordinates in the following manner:

$$\left.\begin{aligned} &\frac{1}{\bar{R}\xi}\frac{\partial}{\partial \xi}\xi q_{\lambda r} - \frac{\bar{a}}{\bar{R}}\xi\frac{\partial}{\partial \xi}q_{\lambda x} + \frac{\partial q_{\lambda x}}{\partial \eta} + \alpha_\lambda c_\lambda U_\lambda = \eta_{c\lambda},\\ &c_\lambda \frac{\partial U_\lambda}{\partial \eta} - \frac{\bar{a}c_\lambda}{\bar{R}}\xi\frac{\partial U_\lambda}{\partial \xi} + 3K_{1\lambda}q_{\lambda x} = 0,\\ &\frac{c_\lambda}{\bar{R}}\frac{\partial U_\lambda}{\partial \xi} + 3K_{1\lambda}q_{\lambda r} = 0, \end{aligned}\right\} \tag{5}$$

where

$$\bar{R} = \frac{f(x)}{R_0}; \quad \bar{a} = \frac{1}{R_0}\frac{df(x)}{dx}.$$

The boundary conditions are also described in the new coordinates.

We can obtain from system (7) an elliptic equation for the radiant energy density U_λ:

$$\left[\frac{1}{\bar{R}^2\xi} + \left(\frac{\bar{a}}{\bar{R}}\right)^2\xi\right]\frac{\partial}{\partial \xi}\frac{\xi}{K_{1\lambda}}\frac{\partial U_\lambda}{\partial \xi} + \frac{\partial}{\partial \eta}\frac{1}{K_{1\lambda}}\frac{\partial U_\lambda}{\partial \eta} - \frac{\bar{a}\xi}{\bar{R}}\left[\frac{\partial}{\partial \xi}\frac{1}{K_{1\lambda}}\frac{\partial U_\lambda}{\partial \eta} + \frac{\partial}{\partial \eta}\frac{1}{K_{1\lambda}}\frac{\partial U_\lambda}{\partial \xi}\right] +$$

$$+ \frac{\xi}{K_{1\lambda}}\left[\left(\frac{\bar{a}}{\bar{R}}\right)^2 - \frac{1}{\bar{R}}\frac{d\bar{a}}{dx}\right]\frac{\partial U_\lambda}{\partial \xi} + \frac{3}{c_\lambda}\left[\eta_{c\lambda} - \alpha_\lambda U_\lambda\right] = 0. \tag{6}$$

It was approximated by second-order finite-difference equations which can be written in the form of a system of three-point vector equations:

$$-A_i \mathbf{U}_{i+1} + B_i \mathbf{U}_i - C_i \mathbf{U}_{i-1} = \mathbf{F}_i. \tag{7}$$

Here $\mathbf{U}_i$ and $\mathbf{F}_i$ are vectors consisting of M elements (M is the number of nodal points along the η axis); A_i, B_i, and C_i are three-diagonal matrices of dimension M × M. The method of matrix trial runs [5] was used to solve the finite-difference equations.

The components of flux density $q_{\lambda r}$ and $q_{\lambda x}$ in the directions of the coordinates r and x are calculated from the values found for $U_{i,j}$ using the second and third equations of system (5).

Let us present some results of our study of the features of the radiation of two-phase flows in Laval nozzles. The experiments were conducted on a sample binary mixture containing particles of Al_2O_3. The composition of the gas phase at the nozzle inlet was as follows, molar fractions: $H_2 = 0.44$; $CO = 0.27$; $HCl = 0.11$; $N_2 = 0.07$; $H_2O = 0.06$; $H = 0.03$; $CO_2 = 0.01$; $Cl = 0.01$. The mass fraction of the condensed phase at the nozzle inlet $\alpha_S = 0.33$. A monotonic increase in particle size along the nozzle axis was assumed. The average-mass radius of the particles $r_{S43} = 1.5\,\mu m$ at the nozzle inlet, $2.5\,\mu m$ in the region of the minimum cross section, and $6\,\mu m$ at the nozzle outlet. The flow stagnation temperature at the nozzle inlet $T_{c0} = 3140°K$, while the flow stagnation pressure $p_{c0} = 4\ MN/m^2$. The Mie solution for diffraction of light on a spherical particle [6, 7] was used to obtain the radiation characteristics of the system of polydispersed condensed-phase particles.

Figure 2 shows the distribution of the flux densities $q_{\lambda R}$ for the radiant flows incident to the nozzle wall (the nozzle contour is shown in the lower part of the figure). The solid lines correspond to the radiant-flow distributions obtained in the two-dimensional formulation. The dashed lines denote the distributions obtained in the unidimensional formulation in the P_3-approximation. We calculated the flux densities in any given cross section of the nozzle in the unidimensional approximation by using the radius of an infinite cylinder and the radiation characteristics of the medium corresponding to this cross section.

It can be seen from Fig. 2 that the flux densities in the converging part of the nozzle are nearly the same according to the one- and two-dimensional approximations. However, in the diverging part of the nozzle, the flux densities calculated in the one-dimensional formulation at short wavelengths are considerably lower than the values obtained in the two-dimensional formulation. The gas phase lacks molecules which absorb effectively at short wavelengths, and the absorption coefficients of the particles of Al_2O_3 condensate are low at the low flow temperatures near the nozzle outlet. As a result, the optical density of the medium decreases and the level of radiation of the two-phase mixture at short wavelengths is determined mainly by radiation from the transonic flow region which is scattered on the particles. In the one-dimensional formulation, the results correspond to the data directly in the cross section being examined. This also helps to explain the sharp differences in the radiant flux obtained in the different approximations. An increase in the wavelength of the radiation is accompanied by an increase in the absorption coefficient of the Al_2O_3 particles and an increase in the importance of gas-phase radiation. In particular, HCl molecule radiation is detected at $\lambda = 4\ \mu m$. This leads to an increase in the optical density of the medium, so that radiation from the higher-temperature zone is absorbed by the layer of two-phase mixture. Thus, in Fig. 2, the distributions of radiant flux incident on the nozzle wall at $\lambda = 4\ \mu m$ coincide in the different approximations.

The calculations show that the effect of radiation from a given point is manifest roughly within the range of 10 units of optical density (Fig. 3). Figure 3 shows the change in the flux densities in the direction of the x axis – $q^+_{\lambda x}$ – and the flux densities for the resulting radiation in this direction. The data correspond to the values on the nozzle axis.

It is shown by the calculations that the flux density of the resulting radiation at the nozzle inlet $q_{\lambda x}$ is nearly zero. This circumstance is connected with the small temperature gradient in the converging part of the nozzle with a medium having a high optical density. A substantial increase in the resulting radiation $q_{\lambda x}$ occurs in the transonic flow region, where there are large gradients of the gasdynamic parameters. The value of $q_{\lambda x}$ reaches a maximum a short distance from the minimum cross section in the direction of the diverging part of the nozzle. The value of $q_{\lambda x}$ gradually decreases downstream, since the radiant flux density decreases in the direction of the x axis. The difference between $q_{\lambda x}$ and $q^+_{\lambda x}$ is small at the nozzle outlet. This shows that radiation from the nozzle direction is significantly greater than radiation from the direction of the free jet, i.e., $q^+_{\lambda x} \gg q^-_{\lambda x}$.

With an increase in the absolute dimensions of the nozzle and in both the gas-absorption lines and bands, the resulting radiation in the direction of the x axis decreases due to an increase in the optical density of the medium.

It can be seen from the completed calculations that the effect of the features of the radiating volume depends on the optical density of the medium. When the optical density of the medium $\tau > 10$, the radiation from the distant zones has little effect on the results, and the calculations can be performed in a unidimensional formulation – particularly in the converging part of the nozzle. At $\tau < 10$, for example, the geometry of the volume in the diverging part of the nozzle has a substantial effect on the radiant flux. In the diverging part, the level of radiation is also affected by the temperature nonuniformity of the gas and condensed phases [1]. Thus, to accurately calculate the radiant flux in supersonic flow regions, it is necessary to use a two-dimensional formulation and to take into account the temperature nonuniformity of the phases and the crystallization kinetics of the condensate particles.

NOTATION

$q_{\lambda r}$, $q_{\lambda x}$, components of radiant flux density in the directions of the coordinate axes r and x; U_λ, radiant energy density; $\eta_{c\lambda}$, volume density of spontaneous radiation; $K_{1\lambda} = \alpha_\lambda + \beta_\lambda(1 - g_1)$, where α_λ is the effective absorption coefficient of the medium; β_λ and g_1, scattering coefficient and mean scattering cosine; c_λ, velocity of the radiation; ε_w and r_w, hemispherical emissivity and reflection coefficient of the bounding surfaces; $I_{\lambda b}(T_w)$, Planck function at the surface temperature T_w; λ, radiation wavelength; $q^+_{\lambda x}$, flux density in the positive direction of the x axis; $q_{\lambda R}$, flux density of radiation incident to nozzle wall; α, angle between r axis and normal to lateral surface of nozzle; r_*, minimum cross section of Laval nozzle; $\bar{r} = r/r_*$, $\bar{x} = x/r_*$, dimensionless coordinates; R_0, minimum cross section of the region of integration; R, coordinates of the lateral boundary of the region over the r axis; L, length of the region of integration over the x axis.

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DIFFUSION SLIP OF A GAS

II. APPLICATION OF THE METHOD OF THE THERMODYNAMICS OF IRREVERSIBLE PROCESSES

S. F. Bakanov, B. V. Deryagin, and V. I. Roldugin

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A method is proposed for thermodynamic calculation of the diffusion slip coefficient.

A system of equations was obtained in [1] to determine small complements to equilibrium (Maxwell) distribution functions for the components of a binary gas mixture flowing slowly in a plane-parallel channel when the temperature and pressure of the gas are held constant. This system was then solved on the assumption that the concentration of one of the components was trivial. This approximation made it possible to convert the system of eight equations into two systems of four equations each, complete the analytical solution to the problem, and calculate the diffusion slip coefficient K_{DS} by directly computing the mean mass velocity of the gas resulting from concentration gradients of the mixture components.

Also of interest is another method of calculating the slip coefficients, based on the use of the methods of the thermodynamics of irreversible processes [2, 3]. Correct realization of this method – apart from a purely formal proof of the directly obtained result – makes it possible to extract important information on the physical nature of the phenomenon and opens up possibilities for experimental measurement of the effect on a new basis.

1. We will examine the problem of the flow of a binary mixture of gases in a plane-parallel channel with a distance 2d between the plates. Let the plates forming the channel be brought into relative motion of a velocity **V** by a force **F**. Given constant pressure and temperature in the channel, if we create a gradient in the concentration of the components of the mixture in the channel, then the total entropy produced in such a system may be written in the form

$$\Delta S = \frac{\mathbf{F}\cdot\mathbf{V}}{T} + k\langle \mathbf{u}_1 - \mathbf{u}_2 \rangle \nabla n_1, \tag{1}$$

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